

Phasor Analysis of GM-type Pulse Tube Refrigerator

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF BACHELOR
OF TECHNOLOGY IN MECHANICAL ENGINEERING



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Certificate of Approval

This is to certify that the thesis entitled '**Phasor Analysis of GM – type Double Inlet Pulse Tube Refrigerator**' submitted by **Manish Kumar** bearing roll number **110ME0297** in the partial fulfillment of the requirement for the degree of Bachelor of Technology in Mechanical Engineering, National Institute of Technology, Rourkela, is being carried out under my supervision.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/institute for the award of any degree or diploma.

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Abstract

The simplicity and varied application of a pulse tube refrigerator has always gained the attention of many scientist and researchers towards itself. It is because of this very reason that various different types of pulse tube refrigerator has come up with time.

A GM type double inlet pulse tube refrigerator (DIPTR) is one of the many configuration of the pulse tube that has come up with time. Although the configuration of a DIPTR seems to be quite simple but its working principle is a bit complicated.

The aim of the present work is to study the working principle behind a DIPTR and develop a MATLAB code for the same and use it further to draw a phasor for the governing equation, so as to validate the output of the code developed.

The MATLAB code and the phasor developed in this work is then used to develop a simplified design method for determining the dimension of the pulse tube of a DIPTR.

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Nomenclature

<u>Symbol</u>	<u>Description</u>
ϵ_v	Porosity of regenerator (\equiv void volume / total volume)
f	Frequency (Hz)
\dot{m} / \dot{M}	Mass flow rate (kg/s)
P	Pressure (MPa)
Q_c	Cooling capacity
R	Gas constant (J/kg K)
T	Temperature (K)
t	Time (s)
V	Volume (m ³)
Z	Orifice impedance (Pa s/kg)
T_{eff}	$\equiv \frac{T_h - T_c}{\ln\left(\frac{T_h}{T_c}\right)}$
V_{eq}	$\equiv V_{chx} + \frac{V_{pt}}{\gamma} + V_{hxx}$
r_v	$\equiv \frac{e_v V_{eq} / V_{pt}}{T_{eff} / T_c}$
h	Enthalpy
U	Internal energy
C_p	Specific heat at constant pressure
C_v	Specific heat at constant volume
C_{reg}	Conductance of regenerator

α	Volumetric fractional pulse tube compliance flow through DI valve
β	Fractional orifice flow though DI valve
ω	Angular frequency
γ	Specific heat ratio ($\equiv c_p/c_v$)
δ	Stokes Boundary layer thickness
ρ	Density
ν	Kinematic viscosity
A	Area of cross section
W	Work

<u>Subscripts</u>	<u>Description</u>
reg	Regenerator
pt	Pulse tube
cp	Compressor
ph	Phase
chx	Cold end heat exchanger
hhx	Hot end heat exchanger
o	Orifice
DI	Double inlet
c	Cold end
h	Hot end
1	Oscillating pressure (MPa)
0	Buffer
amp	Amplitude
< >	Average

Chapter-1

Introduction

Cryogenics comes from the combination of two different Greek words, namely “kryos”, which means very cold or freezing and “genes” means to produce. Cryogenics is thus defined as the branch of physics and engineering which deals with the study of very low temperature (below 123K), their production and the materials behavior at such low temperature.

1.1. Cryocooler

Cryocoolers are refrigeration machines/equipment having very low achievable refrigeration temperature (below 123K) and low refrigeration power (in the order of 5-500 Watts).

1.2. Classification of Cryocooler

Walker in 1983 classified cryocoolers on the basis of type of heat exchanger used into two types [1]:

1.2.1. Recuperative Cryocooler

The flow of the working fluid in this type of cryocooler is unique and hence they are analogous to direct current electrical systems. The compressor and expander have separate inlet and outlet valves for maintaining the flow direction. In rotary motion of components there's a maximum chance for back flow because of which valves are necessary when the system has any rotary or turbine component [2]. The efficiency of the cryocooler depends a lot on the working fluid because it forms an important part of the cycle. The main advantage of recuperative cryocooler is that, that they can be scaled to any size for specific output. Joule Thomson cryocooler and Brayton cryocooler are few of the examples of recuperative type cryocooler.

1.2.2. Regenerative Cryocooler

The flow of working fluid in this type of cryocooler is oscillatory and hence have an analogy to alternative current electrical system. The working fluid inside this type of cryocooler oscillates in cycles and while passing through the regenerator exchanges heat with the wire mesh present within the regenerator. The

regenerator takes up the heat from the working fluid in one half on the cycle and returns the same in the other half. The wire mesh used in regenerator are very efficient because of their very high heat capacity and low heat transfer losses, but these cryocoolers cannot be scaled up to large sizes. The phase relation between mass flow and pressure variation is responsible for the cooling effect produced. The oscillating pressure can be produced with or without the help of valves as in Stirling and Pulse type type cryocooler, and Gifford McMahon type cryocooler respectively.

1.3. Application of Cryocooler

The major applications of cryocoolers are summarized below [3].

1.3.1. Military

- (i) Infrared sensors for night vision & missile guidance
- (ii) Infrared sensors for satellite based surveillance
- (iii) Gamma-ray sensors for monitoring nuclear activity
- (iv) Superconducting magnets used in mine sweeping

1.3.2. Environmental

- (i) Infrared sensors used in satellites for atmospheric studies
- (ii) Pollution monitoring infrared sensors

1.3.3. Commercial

- (i) Cryopumps for semiconductor fabrication
- (ii) Cellular-phone base stations using superconductors
- (iii) Superconductors used in voltage standards
- (iv) Superconductors used in high-speed communications

- (v) Semiconductors used in high-speed computers
- (vi) Infrared sensors employed in NDE and process monitoring
- (vii) Industrial gas liquefaction

1.3.4. Medical

- (i) Cooling of superconducting magnets used in MRI
- (ii) SQUID magnetometers for heart and brain studies
- (iii) Liquefaction of oxygen
- (iv) Cryogenic cryosurgery and catheters

1.3.5. Transportation

- (i) LNG for fleet vehicles
- (ii) Superconducting magnets used in maglev trains
- (iii) Infrared sensors used in aircraft's night vision

1.3.6. Energy

- (i) LNG for peak shaving
- (ii) Superconducting power applications like motors, transformers etc.
- (iii) Thermal loss measurement's infrared sensors

1.3.7. Police and Security

- (i) Infrared sensors used in night-security and rescue

1.3.8. Agriculture

- (i) Storage of biological cells and specimens

Because of the various special application of the cryocooler as mentioned above, the demands for high performance reliability, low vibration, efficiency, long life time, small

size and weight have become an important aspect for the improvement of the cryocoolers. The regenerative cryocoolers have higher efficiency than that of recuperative cryocoolers due to smaller heat transfer loss, both Stirling cryocoolers and Gifford-McMahon (G-M) cryocoolers have an expansion devices (i.e., moving parts) at their cold ends. The moving parts in the cold end are needed in order to adjust the phase angle and to recover the energy flow, which result in the decrease in reliability of the system and shorten the life times of the cryocoolers. The absence of such moving parts in the pulse tube refrigerator/cryocooler at their cold end and thus have an advantages over other cryocoolers due to its simplicity and is hence more reliable in operation.

1.4. Pulse Tube Refrigerator/Cryocooler

Cooling effect at one end of a hollow tube with a pulsating pressure at the other end was first observed by Gifford and Longworth [4] in the early 1960's and marked the inception of one of the most promising cryogenics refrigerators called BPTR i.e. Basic Pulse Tube Refrigerator. The absence of moving parts at the cold end is what that differentiate it from other cryocooler. The associated advantage of simplicity and enhanced reliability has seeked the attention of many research workers and had made it one of the most important topics of modern cryogenics.

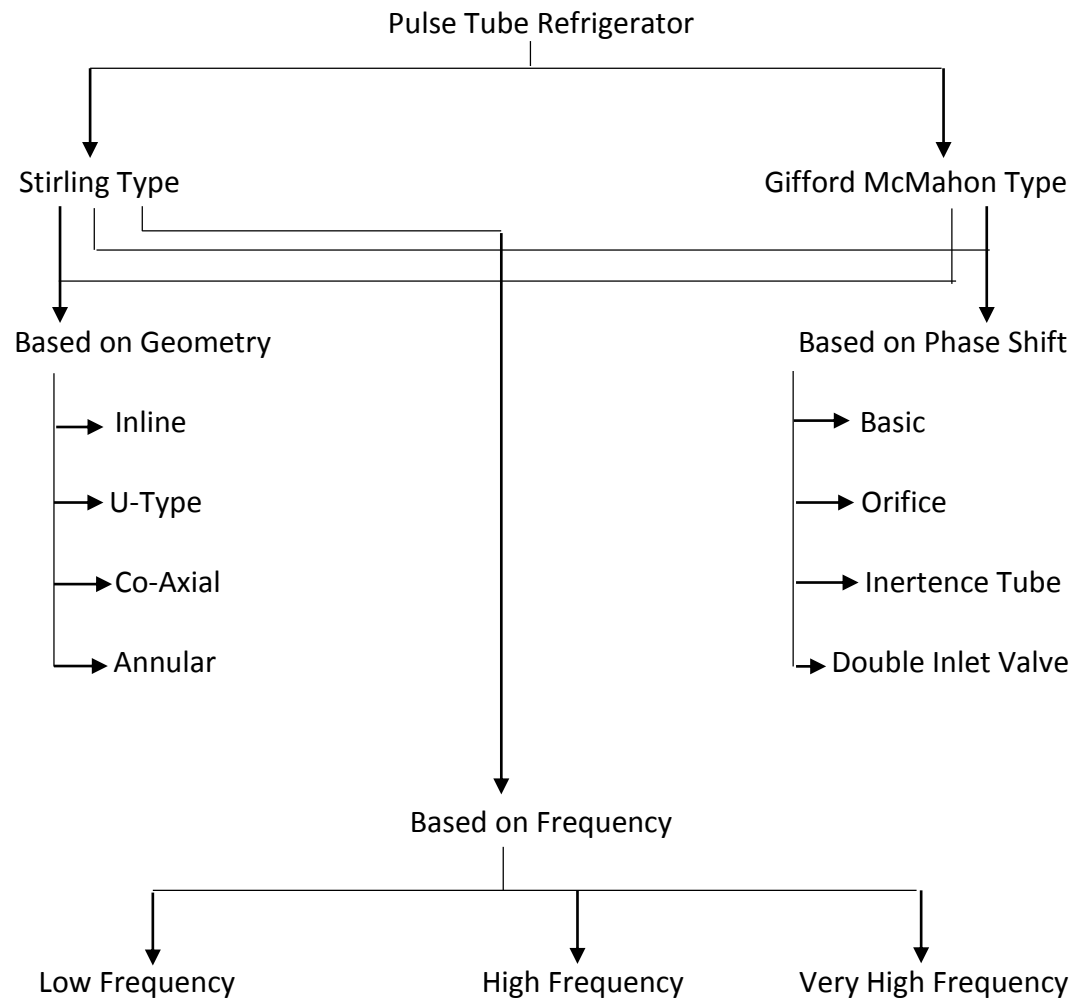
1.4.1. Working of Pulse Tube Refrigerator/Cryocooler

Pulse Tube Refrigerator (PTR) are capable of cooling to a temperature below 123K. The Pulse Tube Refrigerator implements the theory of oscillatory compression and expansion of the gases within a closed volume which is far different from the one used in ordinary refrigeration cycles where the entire refrigeration cycle is based on vapour compression cycles. A PTR is requires a time dependent solution because of the oscillatory movement of the working fluid within it and its because of the same reason that the system at any point in a cycle will reach the same state in the next subsequent cycle.

It is a closed system wherein the oscillating gas which flows throughout the system is generated at one end, which is usually produced by an oscillating piston. The oscillating gas flow can carry away heat from a low temperature point to the hot end heat exchanger if power factor for the phasor quantities are favorable. The size of the pulse tube and the power input determines the maximum amount of heat they can remove.

1.4.2. Classification of Pulse Tube Refrigerator

The following flow chart describes the various types of Pulse Tube Refrigerator with the diagram of the Basic Pulse Tube Refrigerator shown below the same.



Flow Chart 1.1 Classification of Pulse Tube Refrigerator

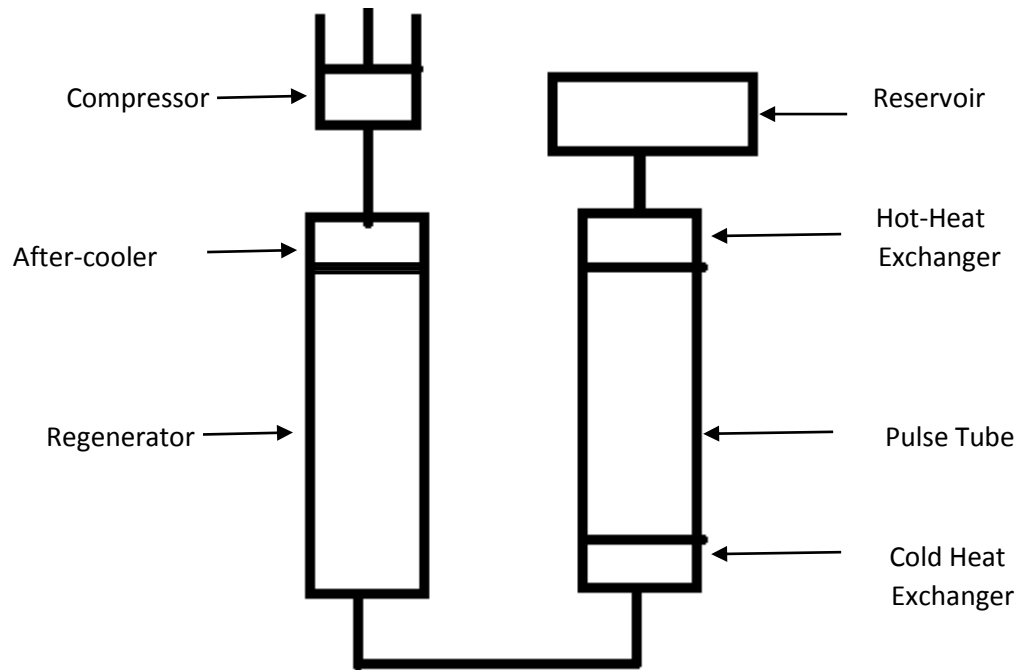


Fig.1.1 Schematic diagram for Stirling Type Orifice Pulse Tube Refrigerator

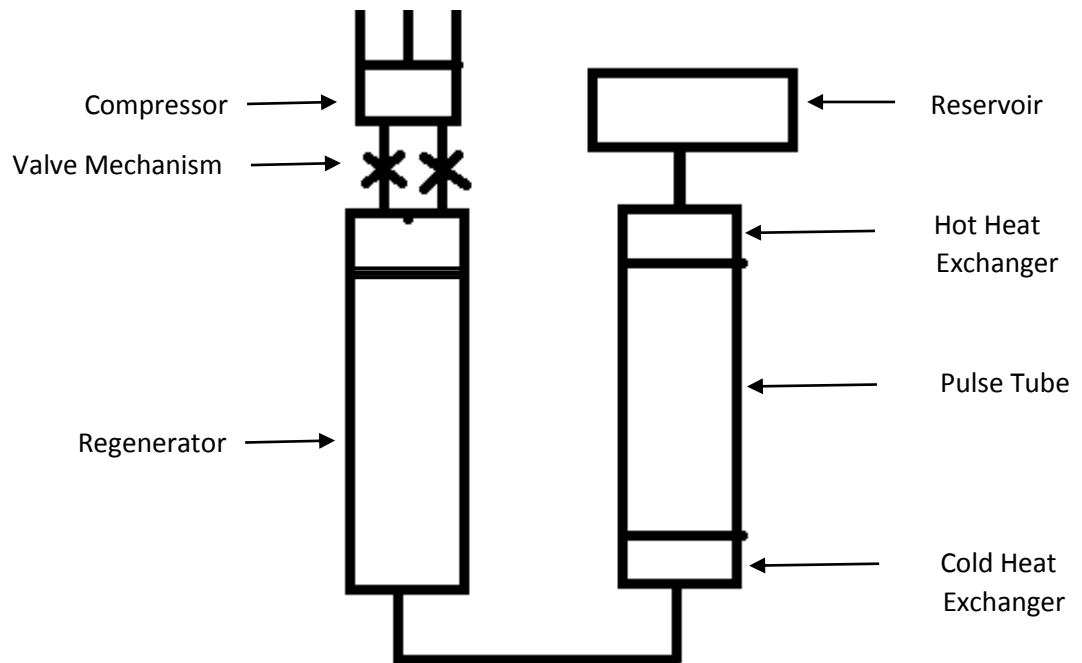


Fig.1.2 Schematic diagram for GM Type Orifice Pulse Tube Refrigerator

1.4.3. Common components of Pulse Tube Refrigerator

1.4.3.1. Compressor

Compressor converts the applied electrical energy into required mechanical input required for compressing the working gas and producing its required oscillation desired in the Pulse Tube Refrigerator.

1.4.3.2. After-Cooler

After-cooler is used to extract the heat from the working gas that it gains due to its compression and thus also helps in reducing the working load of the regenerator.

1.4.3.3. Regenerator

The regenerator is considered to be one of the most important part of the cryocooler, because it's the part of the refrigerator which is responsible for repeatedly removing and giving back heat to the working gas during its oscillation.

1.4.3.4. Cold End Heat Exchanger

It can be viewed as the analogous of the evaporator used in the vapor compression refrigeration cycle. This is where the refrigeration load is absorbed by the system.

1.4.3.5. Pulse Tube

It appears as a hollow tube but is the most critical component of the pulse tube refrigeration system as it's the one which is responsible for transferring heat from the cold heat exchanger to the hot heat exchanger by the enthalpy flow.

1.4.3.6. Hot End Heat Exchanger

Heat of compression in every periodic cycle is rejected through this heat exchanger.

1.5. Double Inlet Pulse Tube Refrigerator (DIPTR)

This configuration of the pulse tube refrigerator incorporates an orifice and a DI (double inlet) valve in the basic pulse tube refrigerator model. The need for employing an orifice and a DI valve was to improve the phase relation between the mass flow rate and the pressure oscillation. The following diagram shows a schematic view of a DIPTR.

Chapter – 2

Literature Survey

In 1963 Gifford and Longworth [4] discovered the Basic Pulse Tube Refrigeration technique where a very simple effect i.e. oscillation of working gas (pressurization and depressurization) makes it possible to construct very low temperature refrigerators without the use of low temperature moving parts or the Joule-Thomson effect. The design was put forward using a hollow tube with one end closed and the other open with the closed end responsible for heat exchange at ambient temperature and the open end serving as the cold end. A thermodynamic model of BPTR was put forward by de Boer [5] with various improvements by taking into account the gas motion during the cooling and heating steps that result in more accurate temperature profiles.

The first improvisation to the basic pulse tube refrigerator was made in 1984 by Mikulin et al. [6] where they installed an orifice and reservoir at the top of the pulse tube to allow some gas to pass into and out of a large reservoir volume. This configuration of the pulse tube refrigerator was given the name as the Orifice Pulse Tube Refrigerator. An analytical model for OPTR was put forward by Starch and Radebaugh [7] who made a simple expression for the gross refrigeration power, which agrees with experiments.

The next improvisation to the pulse tube was made by Zhu et al. [8], where they introduced a double inlet valve in the orifice pulse tube model and thus named their configuration of the pulse tube as Double Inlet Pulse Tube Refrigerator (DIPTR). The reason for introducing a double inlet valve along with the orifice was to optimize the phase relation between the mass flow rate and the pressure oscillation. An improved numerical model for simulating the oscillating fluid flow and detail dynamic performance of the OPTR and DIPTR was put forward by Ju et al.[9]

Because of the simplicity of the mechanism employed in here and the lower attainable temperature, pulse tube refrigerator has caught eyes of many research workers. A lot of work are being done to understand its working principle and a number of theories have been put forward to explain the same.

Prakash in [10] came out with the theoretical analysis of the thermodynamics equations governing the mechanism behind the double inlet pulse tube refrigerator. He plotted the various mass flow rate graphs and the pressure oscillation graph using software. The results obtained

from his work resembles a lot to the experimental data but there were still fluctuation in the results obtained by his work.

In the later works, the mass flow rates were made analogous to AC current flow. This concept led to phasor representation of the mass flow rates and the pressure wave. L.Mohanta and M.D. Atrey in 2011 [11] came forward with one such phasor diagram representing the various mass flow rates. This phasor representation of the mass flow rate became a prominent way of analyzing the phase difference between the mass flow rates and the pressure variation.

Hoffman and Pan [12] studied the phase shifting in Pulse Tube Refrigerator and worked on the phasor representation of the mass flow rates and pressure oscillation. They studied the phase relation for different configuration of the pulse tube refrigerators and experimentally concluded the optimum phase relation for the same.

These phasors represented each mass flow rates as a vector quantity and just plotted the governing equations. There were no concrete relation as to how the exact phase difference can be obtained. They merely served as a method to cross verify the experimental/analytical works.

Chapter – 3

Aim Of The Present Work

After going through the various literature, we decided to work on the following topics related to Double Inlet Pulse Refrigerator (DIPTR):

- 3.1. To study the thermodynamic phenomenon occurring within the DIPTR and derive the equations for various mass flow rates and the pressure variation.
- 3.2. To develop a MATLAB code from the governing thermodynamic equations, so as to get the exact variation of mass flow rates and the pressure oscillation within the DIPTR.
- 3.3. To plot the mass flow rates and pressure oscillation on a phasor, so as to visualize the dependence of one quantity on the other and study the phase relationship.
- 3.4. To design the dimension of a pulse tube based on the COP and the refrigeration effect required.

Chapter – 4

Thermodynamic Study of Double Inlet Pulse Tube Refrigerator (DIPTR)

The study of the working process of the pulse tube refrigerator becomes very complex due to the oscillating flow and due to the presence of the regenerator, orifice-reservoir and the double inlet valve. Compression and expansion of the gas column inside the pulse tube is the reason behind the cooling effect observed at the cold end of the pulse tube. The compression and expansion process of the working gas within the pulse tube lies between adiabatic and isothermal processes.

Liang et al [13] was the first to attempt solving the working mechanism of pulse tube refrigerator by analyzing the thermodynamic behavior of the gas element as adiabatic process.

The following assumptions are made in conjunction with the adiabatic behavior of the working gas:

- Hot-end heat exchanger, cold-end heat exchanger and the regenerator have been assumed to be perfect, which means that there will be a constant temperature gradient between its hot end and its cold end and the heat exchangers will work at constant temperature at steady state.
- Working fluid has been assumed to be an ideal gas.
- Viscous effect of the gas has been neglected.
- The variation of mass flow rates, pressure and temperature has been assumed to be sinusoidal.
- There is no phase difference between the pressure and the temperature throughout the working space of the pulse tube refrigerator.
- There is no length wise mixing or heat conduction.

The following figure represents a GM type Double Inlet Pulse Tube Refrigerator (DIPTR) with the working fluid assumed to be Helium gas.

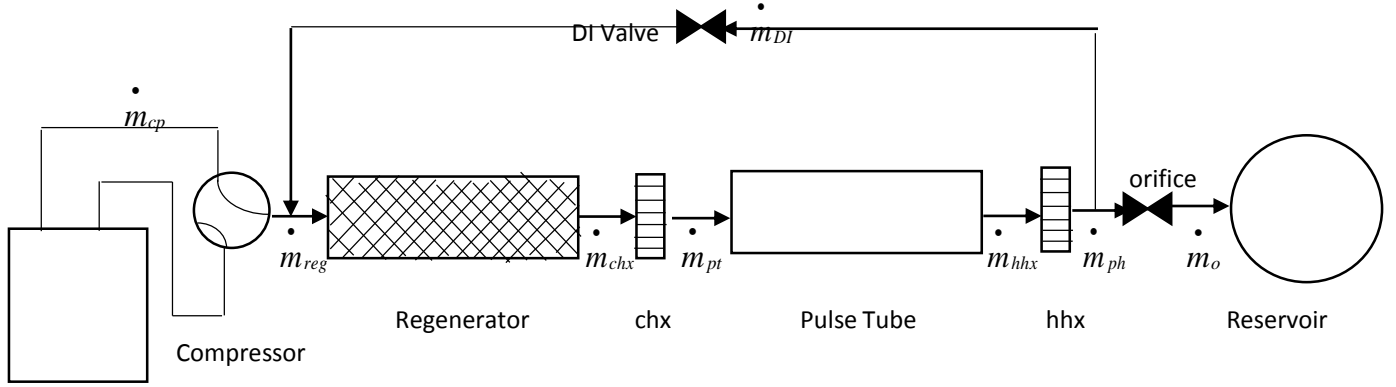


Fig. 4.1 Schematic Diagram of a Double Inlet Pulse Tube Refrigerator (DIPTR)

The pressure variation within the pulse tube has been assumed to be sinusoidal, so the pressure variation at any instant within the pulse tube is computed with the help of the following equation, i.e.

$$P_{pt} = P_0 + P_1 \sin(\omega t) \quad \dots(1)$$

where;

$$\omega = 2 \pi f \quad \dots(2)$$

Now in order to calculate mass flow rate, pressure and temperature as a function of time and position in the system, the governing equations are applied to all of the discrete volumes. These equations include the ideal gas law, the mass conservation equation and the energy conservation equations.

Substituting the ideal gas law into the mass conservation equations for the regenerator gives [14]:

$$\left(\frac{dM}{dt} \right)_{reg} = \frac{d}{dx} \left(\frac{PV_{reg}}{RT_{eff}} \right) = \frac{V_{reg}}{RT_{eff}} \frac{dP}{dt} = \dot{m}_{reg} - \dot{m}_{chx} \quad \dots(3)$$

where;

$$T_{eff} = \frac{T_h - T_c}{\ln(T_h / T_c)} \quad \dots(4)$$

As the temperature profile within the regenerator has been assumed to vary linearly along its length, so instead of average temperature we have to take the logarithmic mean temperature of the same.

Since the temperature at the hot-end heat exchanger and the cold-end heat exchanger has been assumed to be constant so similarly proceeding we can get the mass flow rates at the hot-end heat exchanger and the cold-end heat exchanger as:

$$\left(\frac{dM}{dt}\right)_{chx} = \frac{d}{dx} \left(\frac{PV_{chx}}{RT_c} \right) = \frac{V_{chx}}{RT_c} \frac{dP}{dt} = \dot{m}_{chx} - \dot{m}_{pt} \quad \dots(5)$$

and

$$\left(\frac{dM}{dt}\right)_{hhx} = \frac{d}{dx} \left(\frac{PV_{hhx}}{RT_h} \right) = \frac{V_{hhx}}{RT_h} \frac{dP}{dt} = \dot{m}_{hhx} - \dot{m}_{ph} \quad \dots(6)$$

For determining the mass flow rate within the pulse tube we assume energy conservation equation instead of mass conservation as the temperature is known to vary along with the pressure which is sinusoidal in nature. Hence applying the energy conservation equation in the pulse tube we get,

$$\left(\frac{dU}{dt}\right)_{pt} = \dot{m}_{pt} h_c - \dot{m}_{hhx} h_h \quad \dots(7)$$

From equation number (7) and the ideal gas law, we get

$$\frac{c_v V_{pt}}{R} \frac{dP}{dt} = c_p \left(\dot{m}_{pt} T_c - \dot{m}_{hhx} T_h \right) \quad \dots(8)$$

or,

$$\dot{m}_{pt} = \frac{V_{pt}}{\gamma R T_c} \frac{dP}{dt} + \frac{T_h}{T_c} \dot{m}_{hhx} \quad \dots(9)$$

Combining equation (5), (6) and (9) we get another way of expressing the mass flow rate through the cold-end heat exchanger, i.e.

$$\dot{m}_{chx} = \frac{V_{eq}}{RT_c} \frac{dP}{dt} + \frac{T_h}{T_c} \dot{m}_{ph} \quad \dots(10)$$

where;

$$V_{eq} = V_{chx} + V_{pt}/\gamma + V_{hcx} \quad \dots(11)$$

In equation (6) and (10), \dot{m}_{ph} is the sum total of mass flow rates through the orifice and the double inlet valve, i.e.

$$\dot{m}_{ph} = \dot{m}_{DI} + \dot{m}_o \quad \dots(12)$$

where the mass flow rate through the orifice is due to the pressure difference prevailing across it and hence can be given by:

$$\dot{m}_o = ((P - P_o)/Z) \quad \dots(13)$$

and, Z is the orifice impendence [12] given by the formula:

$$Z = \frac{\gamma P_0}{V_0} \quad \dots(14)$$

In equation (12), the mass flow rate through the double inlet valve is the sum of the pulse tube compliance flow through the orifice and the hot-end heat exchanger. The mass flow rate through the double inlet valve can be found by introducing the double inlet factor α [15] as:

$$\dot{m}_{DI} = -\frac{\alpha V_{eq}}{RT_h} \frac{dP}{dt} - \beta \dot{m}_o \quad \dots(15)$$

where α and β are the volumetric fractional pulse tube compliance mass flow rate through double inlet valve and fractional orifice mass flow rate through the double inlet valve respectively. The mathematical form of β is given by Zhu et al [15] and that of α can be found by assuming that the

ratio of the mass flow rate through the regenerator and the double inlet valve to be in a constant ratio [16], i.e.

$$\beta = \frac{1}{k} \times \frac{1}{\frac{T_h}{T_c} \times \frac{V_{reg}}{V_{pt}}} \times \frac{\left(\frac{T_h}{T_c} + 1\right) \times \left(\frac{T_h}{T_c} - 1\right)^2}{\left(\frac{T_h}{T_c}\right)^2 \ln\left(\frac{T_h}{T_c}\right) - \frac{1}{2}\left(\frac{T_h}{T_c}\right)^2 + \frac{1}{2}} \frac{\gamma P_0}{V_0} \quad \dots(16)$$

and

$$\alpha = \beta \left(1 + e_v \frac{V_{reg}}{V_{eq}} \frac{T_c}{T_{eff}}\right) \quad \dots(17)$$

The pressure variation within the pulse tube is already know but the pressure variation of the compressor is still not known, which can be found by assuming that the mass flow rate within the regenerator is directly proportional to the pressure difference between the compressor and the pulse tube i.e.

$$\dot{m}_{rg} = C_{reg} (P_{cp} - P_t) \quad \dots(18)$$

where, C_{rg} is calculated using Ergun's law for laminar flow [17] and is mathematically given by the following formula:

$$C_{reg} = \frac{\rho \pi D_{reg}^2 D_h^2}{4 \times 150 L_{reg} \mu} \frac{e_v^3}{(1 - e_v)^2} \quad \dots(19)$$

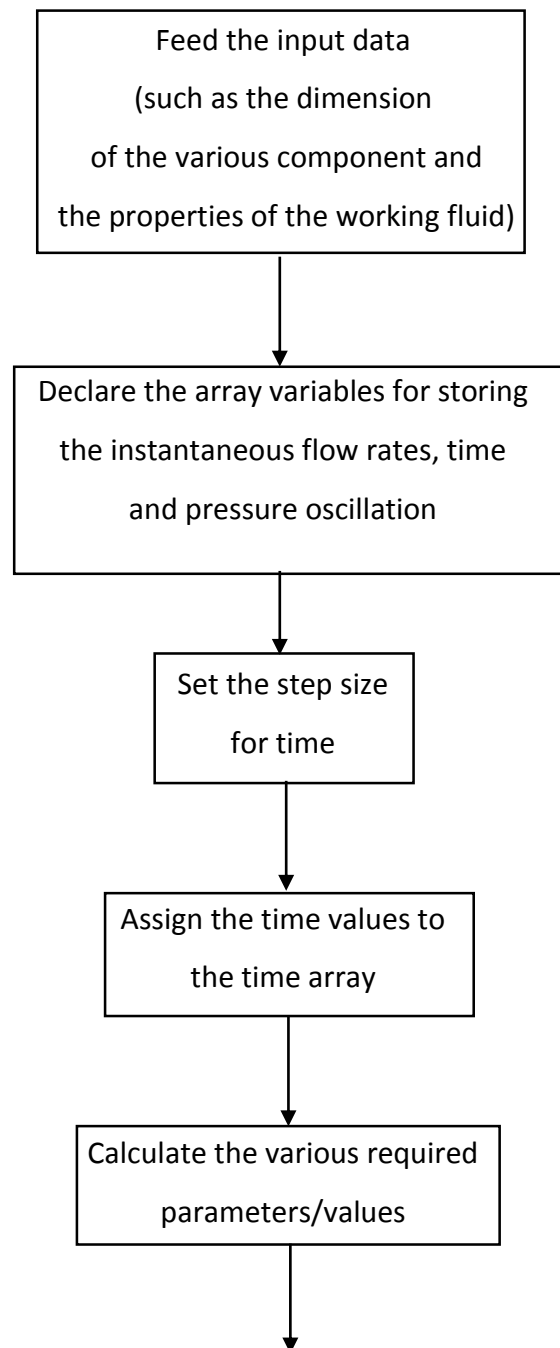
As shown in the schematic diagram, it is clear that the total mass flow rate through the compressor will be the sum of the mass flow rates through the regenerator and the double inlet valve, thus we have;

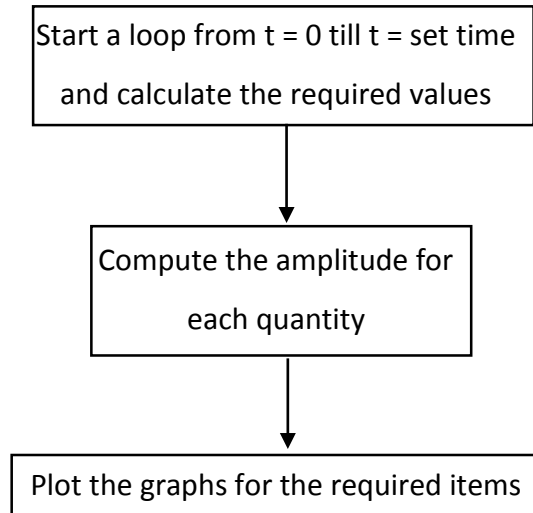
$$\dot{m}_{cp} = \dot{m}_{reg} + \dot{m}_{DI} \quad \dots(20)$$

Chapter – 5

Algorithm for the Program and Output

The algorithm for the above set of equations was made and the concerned program was written in MATLAB so as to get the exact mass flow rates, the pressure variation and the different phase relations required for plotting the phasor. The following flow chart shows the methodology adopted for writing the code.





Flow Chart 5.1 Flow Chart for the MATLAB program

The above algorithm was compiled in MATLAB program and was executed for the following configuration of the DIPTR taken from [10]:

a) Design Data

Table 5.1 Design Data for Adiabatic Model

<u>Components</u>	<u>Parameters</u>
Regenerator	Length (L_{reg}) = 0.3 m Diameter (D_{reg}) = 0.032 m Porosity (e_v) = 0.7 Hydraulic Diameter (D_h) = 0.04 mm
Pulse Tube	Length (L_{pt}) = 0.8 m Diameter (D_{pt}) = 0.02 m Volume (V_{pt}) = 0.00025 m ³
Cold-end Heat Exchanger	Dead Volume (V_{chx}) = 0.00002 m ³
Hot-end Heat Exchanger	Dead Volume (V_{hhx}) = 0.00002 m ³
Orifice	Diameter (D_0) = 1 mm
DI Valve	Diameter (D_0) = 1 mm
Reservoir	Volume (V_r) = 0.007 m ³

b) Operating Condition

Table 5.2 Operating Condition for Adiabatic Model

<u>Operating Parameters</u>	<u>Values</u>
Average Pressure	10.5 bar
Oscillating Pressure	2 bar
Frequency	2 Hz
Cold- End Heat Exchanger	100 K
Hot-End Heat Exchanger	300 K

c) Fluid Data for Helium

Table 5.3 Fluid Data for Adiabatic Model

<u>Physical Condition</u>	<u>Physical Properties</u>
Temperature (200K) Pressure (10 bar)	Dynamic Viscosity (μ) = 15.21×10^{-6} Ns/m ² Density (ρ) = 2.389 Kg/m ³ Specific Heat Capacity at Constant Pressure (C_p) = 5193.0 J/Kg K Gas Constant (R) = 2074.6 J/Kg K Adiabatic Constant (γ) = 1.67

The following results were obtained from the execution of the MATLAB code which were in accordance to the results obtained by Prakash [10] with the graphs being more smoother than what obtained by his analysis:

- A. Graph representing the pressure variation within the pulse tube (P_t), the compressor (P_c) and the reservoir (P_r).

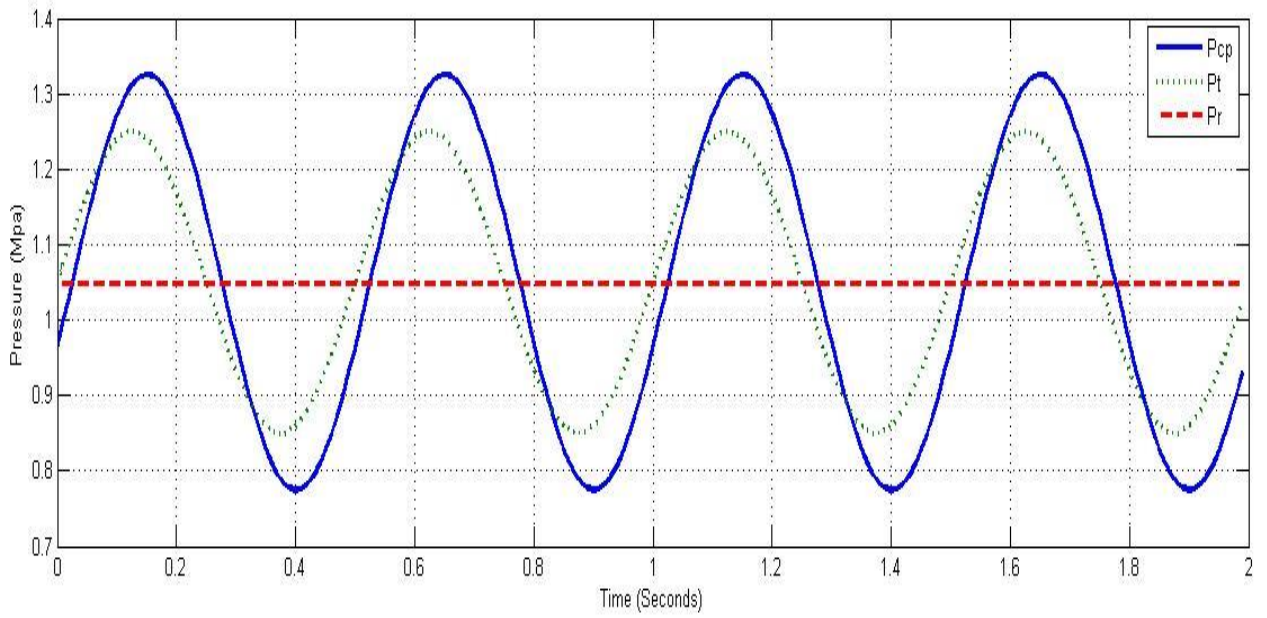


Fig 5.1 Graph representing the pressure variation within the pulse tube (Pt), the compressor (Pc) and the reservoir (Pr)

B. Graph representing the mass flow rates through the regenerator $\left(\dot{M}_{reg}\right)$, double inlet

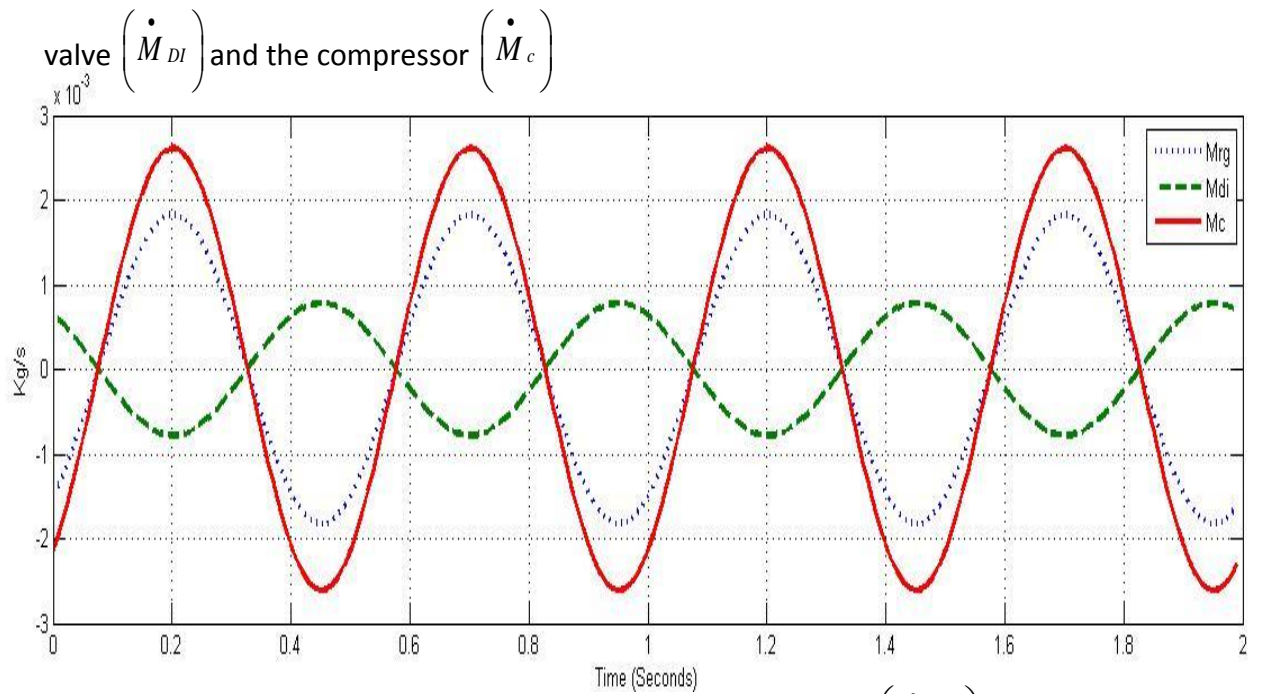


Fig 5.2 Graph representing the mass flow rates through the regenerator $\left(\dot{M}_{reg}\right)$, double inlet valve

$\left(\dot{M}_{DI}\right)$ and the compressor $\left(\dot{M}_c\right)$

C. Graph representing the mass flow rates through hot-end heat exchanger $\left(\dot{M}_{hhx}\right)$, the

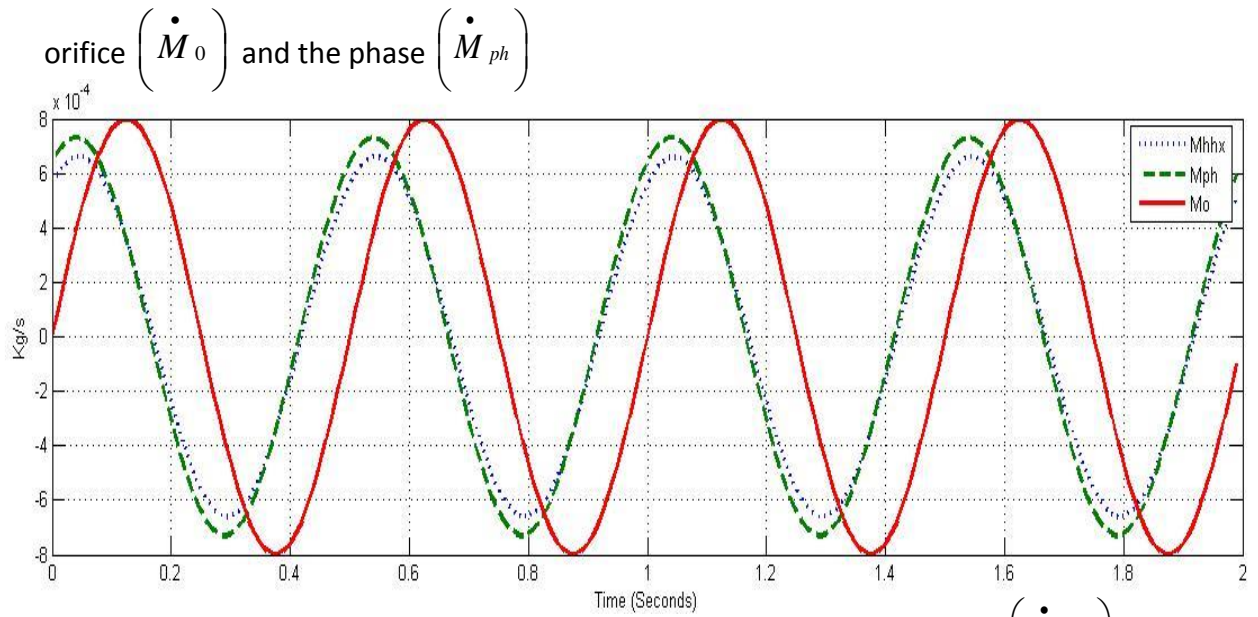


Fig 5.3 Graph representing the mass flow rates through hot-end heat exchanger $\left(\dot{M}_{hhx}\right)$, the

orifice $\left(\dot{M}_o\right)$ and the phase $\left(\dot{M}_{ph}\right)$

D. Graph representing mass flow rates through the compressor $\left(\dot{M}_{pt}\right)$ and the pressure

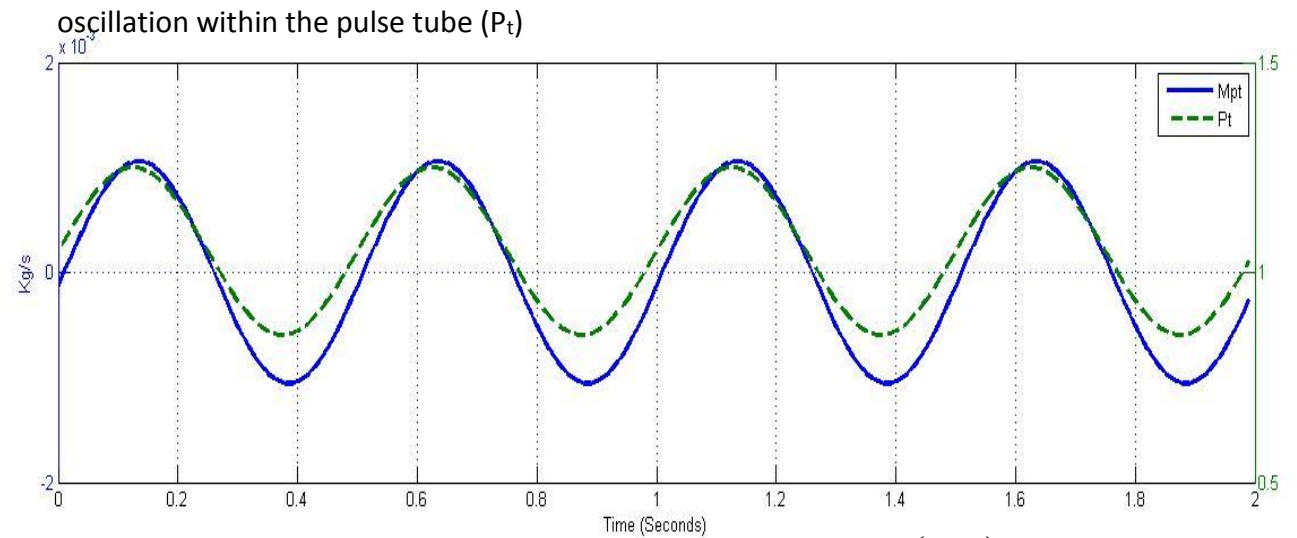


Fig 5.4 Graph representing mass flow rates through the compressor $\left(\dot{M}_{pt}\right)$ and the pressure oscillation within the pulse tube (Pt)

E. Graph representing mass flow rate through cold-end heat exchanger $\left(\dot{M}_{chx}\right)$ and the

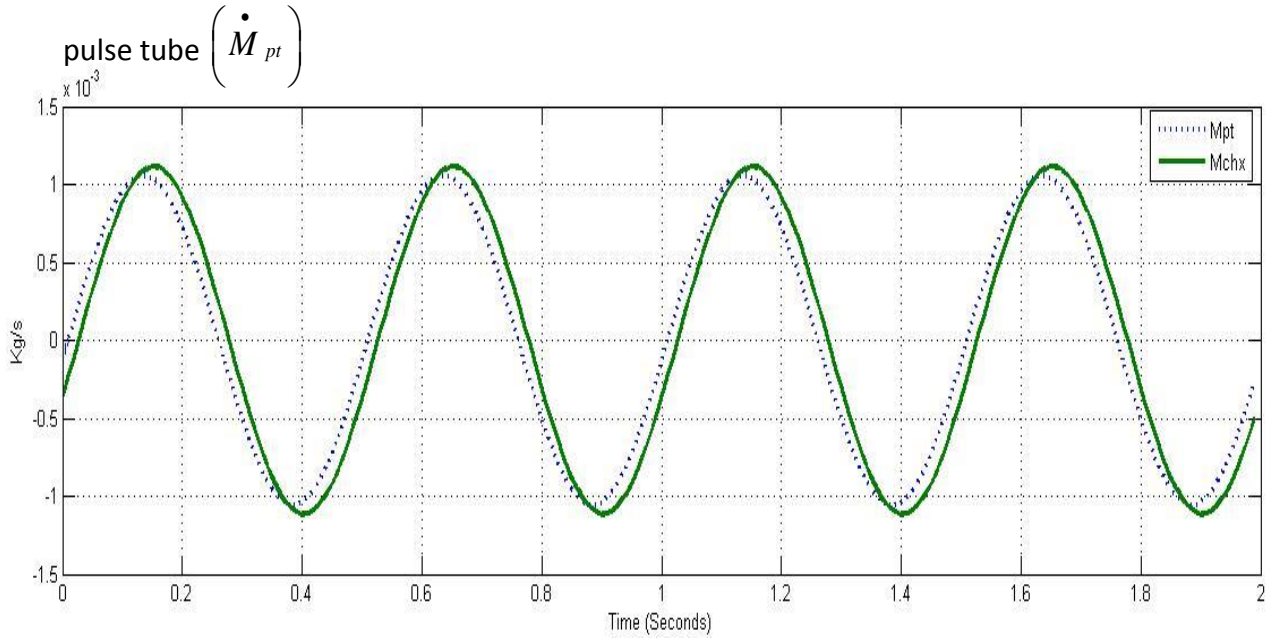


Fig 5.5 Graph representing mass flow rate through cold-end heat exchanger $\left(\dot{M}_{chx}\right)$ and the pulse tube $\left(\dot{M}_{pt}\right)$

The amplitude of the various quantities like the mass flow rates through the various parts of the DIPTR and the pressure amplitude seen at the compressor and the pulse tube were also obtained from the above program. The values are enlisted below which were then later used to draw the phasor diagram.

Table 5.4 Output of the MATLAB code

<u>Parameter</u>	<u>Amplitude</u>	<u>Mean Line</u>
Mass flow rate through orifice	7.9683e-004 kg/s	0 kg/s
Mass flow rate through hot-end heat exchanger	6.5950e-004 kg/s	0 kg/s

Mass flow rate through cold-end heat exchanger	0.0011 kg/s	0 kg/s
Mass flow rate through the pulse tube	0.0011 kg/s	0 kg/s
Mass flow rate through the regenerator	0.0018 kg/s	0 kg/s
Mass flow rate through the double inlet valve	7.8245e-004 kg/s	0 kg/s
Mass flow rate through the compressor	0.0026 kg/s	0 kg/s
Mass flow rate through the phase	7.3027e-004 kg/s	0 kg/s
Pressure amplitude in the pulse tube	0.2 MPa	1.05 MPa
Pressure amplitude in the compressor	0.27555 MPa	1.05 MPa

Chapter – 6

Phasor Analysis

Phasor or phase vector is a way of representation of a sinusoidal function whose amplitude (A), phase (θ) and frequency (ω) are time-invariant. It can be called as a subset of a more general concept called analytic representation. It separate the dependencies on A, θ and ω into three independent factors. This is particularly useful because the frequency factor (which is the time-dependence of the sin or cosine plots) is often common to all the components of a linear combination of sinusoids and phasors allow this common feature to be factored out, leaving just the phase and amplitude which results in conversion of trigonometry and linear differential equations into an algebraic ones.

Phasor analysis is one of the easiest method to verify the consistency of the vectorial equations and hence recheck the obtained results. Phasor analysis for the performed by Hofman and Pan [12] in 1999, by MD Atrey [11] in 2011 and many others but all of them assumed the analogy of electrical circuits in the pulse tube model and none of them gave an exact procedure as how to obtain the phasor for a DIPTR. They all plotted the phasor for the equations with the assumption that every quantity in the above foresaid equation represents a vector and that the mass flow rate through the orifice and the double inlet valve due to the pressure drop across it can be assumed to be an analogy to electric current flowing through a potential drop.

Until in 2007, Jehon and Jeong [16] gave a relationship between the mass flow rate between the orifice and the double inlet valve for a DIPTR. The same relationship has been employed in the above program and the phasor is plotted for the same using the amplitude of the various parameters obtained and enlisted in table 5.4. The following diagram shows the phasor for the above specified dimension of the DIPTR with the symbols having their usual meaning and the scale used is:

$$1 \text{ kg/s} = 50000 \text{ mm}$$

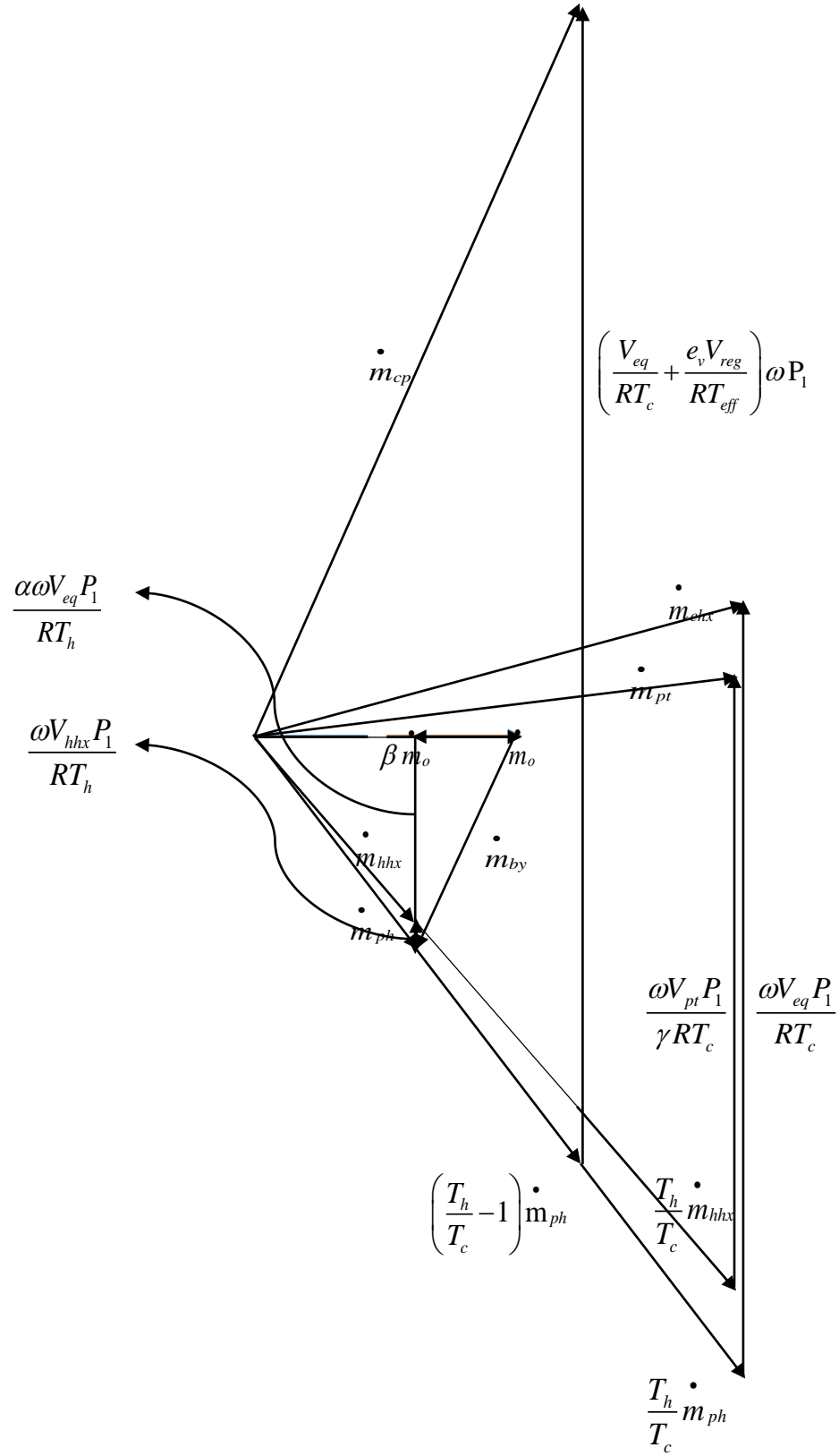


Fig.6.1 Phasor Representation for a Double Inlet Pulse Tube Refrigerator (DIPTR)

Chapter – 7

Design of the Pulse Tube

There are various ways in which the design for a pulse can be done. Some used the entropy method [16] where as some used the software like REGEN [18] for finding out the desired dimension of the pulse tube for a set working condition.

In this present work we came up with a new and simplified methodology for finding out the required dimensions of a pulse tube working at a specified condition. The steps for the same is discussed below, which is later verified using the output of the MATLAB code:

7.1. Steps for Designing

7.1.1. Set the working parameters

We first set the desired working parameters for a DIPTR such as the mean pressure (P_0), the oscillating pressure (P_1), the refrigeration power required (Q_c), working frequency (f), an orifice with known max mass flow rate through it (\dot{m}_o) and the operating temperatures (T_h and T_c)

In the present work we have decided to design a pulse tube for a DIPTR whose working parameters are:

Table 7.1 Input variables for the design

<u>Parameter</u>	<u>Symbol</u>	<u>Data</u>
Mean Pressure	P_0	20 MPa
Oscillating Pressure	P_1	2 MPa
Refrigeration Power Required	Q_c	50 Watts
Working Frequency	f	2 Hz
Cold end temperature	T_c	60 K
Hot end temperature	T_h	300 K
Mass flow through orifice	\dot{m}_o	4.18×10^{-4} kg/s

7.1.2. COP of the Double Inlet Pulse Tube Refrigerator (DIPTR)

The COP (co-efficient of performance) for a standard Carnot refrigerator is given by the following formula:

$$(COP)_{carnot} = \frac{T_c}{T_h - T_c} \quad \dots(21)$$

Since the COP of a GM type pulse tube refrigerator is about 15%-20% of the standard Carnot refrigerator [19]. So we have:

$$(COP)_{GM_pulse_tube} = 0.15(COP)_{carnot} \quad \dots(22)$$

The COP of a refrigerator is actually the ratio of the cooling effect produced by the system to the work input, hence we can define COP in another way as:

$$COP = \frac{Q_c}{W_{cp}} \quad \dots(23)$$

So for the set model we get the value of W_{cp} to be as 534Watts

7.1.3. Mass flow rate through the compressor

The average mass flow rate through the compressor can be found by using the relation [16]:

$$W_{comp} = \frac{\gamma}{\gamma - 1} \langle \dot{m}_{cp} \rangle C_p \frac{T_h}{P_0} (P_H - P_L) \quad \dots(24)$$

Where $\langle \dot{m}_{cp} \rangle$ is the average mass flow rate through the compressor which is related to its amplitude by:

$$\langle \dot{m}_{cp} \rangle = \frac{2(\dot{m}_{cp})_{amp}}{\pi} \quad \dots(25)$$

Hence we get the value of the amplitude of the mass flow rate through the compressor to be as 6.72×10^{-3} kg/s

7.1.4. Volume ratio between the regenerator and the pulse tube

The phasor diagram is then used to find a relation between the mass flow rates through the orifice and the compressor. The obtained relationship is:

$$\left(\dot{m}_{cp} \right)_{amp} = \left(\frac{e_v V_{reg}}{RT_{eff}} + \frac{V_{eq}}{RT_c} + \left(\frac{T_h}{T_c} - 1 \right) \left(\frac{-\alpha V_{eq}}{RT_h} \right) \right) \omega P_1 + \left(\frac{T_h}{T_c} - \beta \left(\frac{T_h}{T_c} - 1 \right) \right) \left(\dot{m}_o \right)_{amp} \quad \dots(26)$$

where $\left(\dot{m}_o \right)_{amp}$ can be found by using equation (13).

The volume of the regenerator and that of the pulse tube can be assumed to have a particular ratio or else we can have the volume of the regenerator using REGEN software [18]. For the present work we have assumed the volume of regenerator and that of the pulse tube to be same, which is true in most of the practical cases. Hence we have:

$$V_{rg} = V_{pt} = 8.034 \times 10^{-5} \text{ m}^3$$

7.1.5. Mass flow rate through the pulse tube

The phasor is again used to find out the amplitude of the mass flow rate through the pulse tube which is given by the formula:

$$\left(\dot{m}_{pt} \right)_{amp} = \left(\frac{V_{pt}}{\gamma RT_c} + \frac{V_{hbx}}{RT_c} - \frac{\alpha V_{eq}}{RT_c} \right) \omega P_1 + \frac{T_h}{T_c} (1 - \beta) \left(\dot{m}_o \right)_{amp} \quad \dots(27)$$

The obtained value for our design is 5.72×10^{-3} Kg/s

7.1.6. Thermal boundary layer thickness

Since we have assumed no heat dissipation across the wall of the pulse tube the diameter of the pulse tube should be large enough to neglect the effect of any

boundary layer formation. The boundary layer thickness can be found out by using the Stokes formula given as:

$$\delta = \sqrt{\frac{2\nu}{\omega}} \quad \dots(28)$$

The value obtained for our design requirement is 3.337×10^{-4} m.

7.1.7. Minimum area of cross section of the pulse tube

We have assumed a laminar flow of the oscillating flow within the pulse tube, hence the Reynolds Number for the same should be within certain limits which is 280 in this case [20]. The fixing up of the Reynolds number makes it easier to calculate the minimum area of cross-section as given by the formula [20]:

$$A_{\min} \geq \frac{\dot{m}_{pt}}{Re \rho v} \quad \dots(29)$$

The minimum area thus obtained from the above relation is 2.12×10^{-4} m.

7.1.8. Dimension of the pulse tube

With the minimum area of cross section known we can find out the minimum diameter for the pulse tube and then take assume a diameter of about 1.2-1.5 times the obtained diameter. The diameter obtained would then be used to calculate the length of the pulse tube as the volume being calculated earlier.

The obtained diameter for the above case is about 1.5 times the calculated diameter and is 2.46 cm and hence the length is 16.3 cm.

7.1.9. Volume of the reservoir

Since we have assumed mass flow through the orifice as given under orifice specification, the volume of the reservoir can be found using the relation given in equation (14), which comes out to be 0.007 m^3

7.2. MATLAB Output

7.2.1. Graphs

- A. Graph representing the pressure oscillation within the pulse tube (P_t), the compressor (P_{cp}) and the reservoir (P_r)

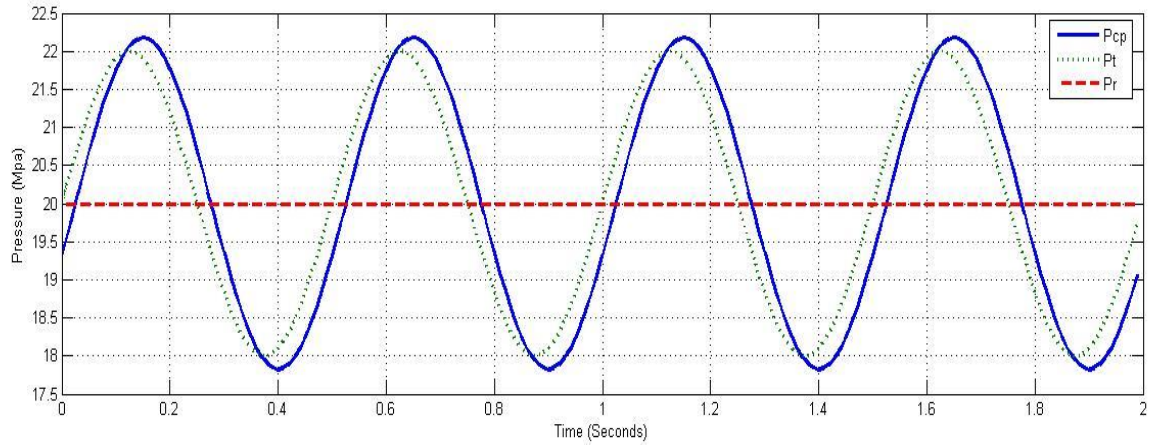


Fig 7.1 Graph representing the pressure oscillation within the pulse tube (P_t), the compressor (P_{cp}) and the reservoir (P_r)

- B. Graph representing the mass flow rates through the regenerator (\dot{M}_{reg}), the

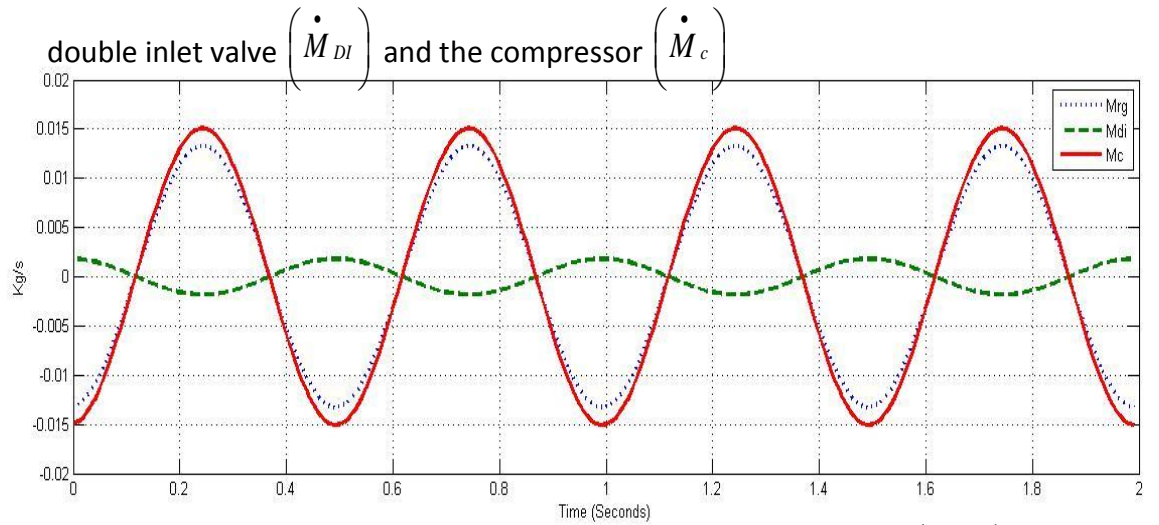


Fig 7.2 Graph representing the mass flow rates through the regenerator (\dot{M}_{reg}), the

double inlet valve (\dot{M}_{DI}) and the compressor (\dot{M}_c)

C. Graph representing the mass flow rate through the hot-end heat exchanger

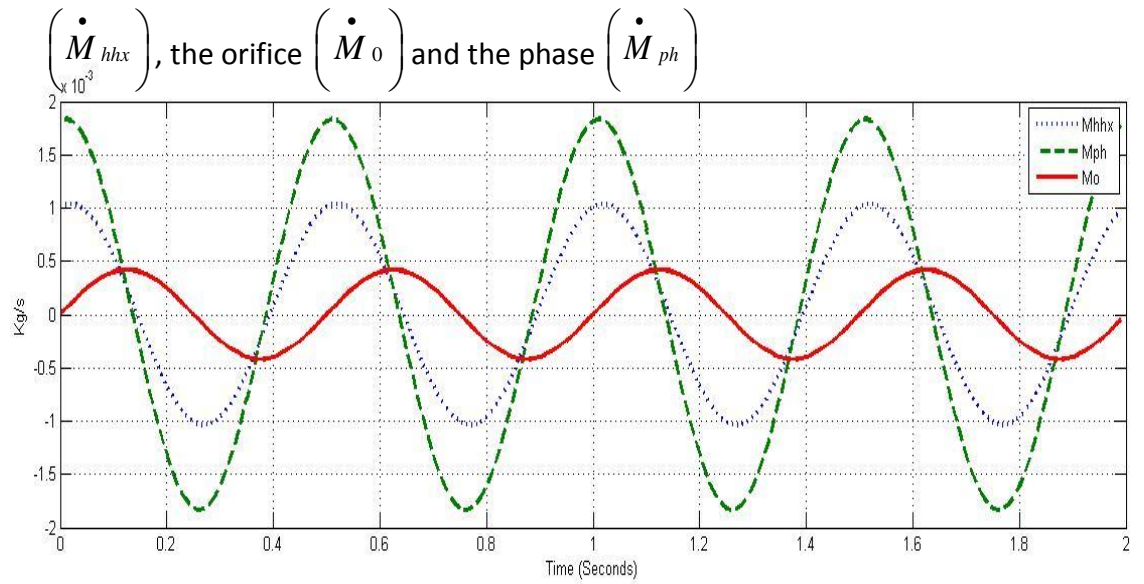


Fig 7.3 Graph representing the mass flow rate through the hot-end heat exchanger

$\left(\dot{M}_{hhx}\right)$, the orifice $\left(\dot{M}_o\right)$ and the phase $\left(\dot{M}_{ph}\right)$

D. Graph representing mass flow rate through the compressor $\left(\dot{M}_{pt}\right)$ and the pressure oscillation within the pulse tube (P_t)

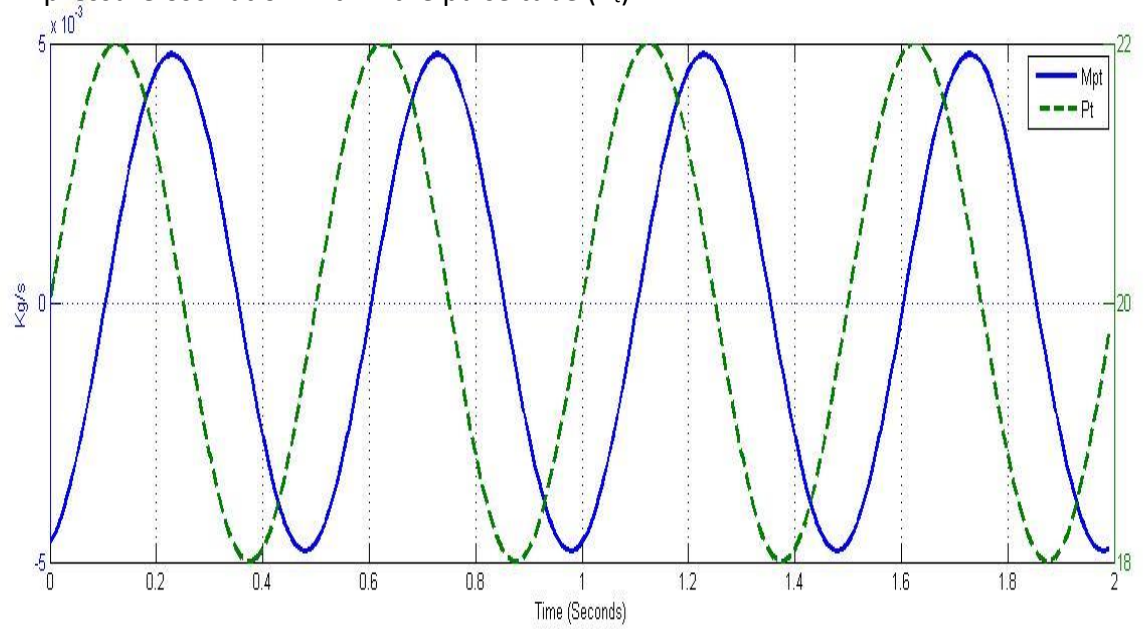


Fig 7.4 Graph representing mass flow rate through the compressor $\left(\dot{M}_{pt}\right)$ and the pressure oscillation within the pulse tube (P_t)

E. Graph representing the mass flow rate through the cold-end heat exchanger $\left(\dot{M}_{chx}\right)$ and the pulse tube $\left(\dot{M}_{pt}\right)$

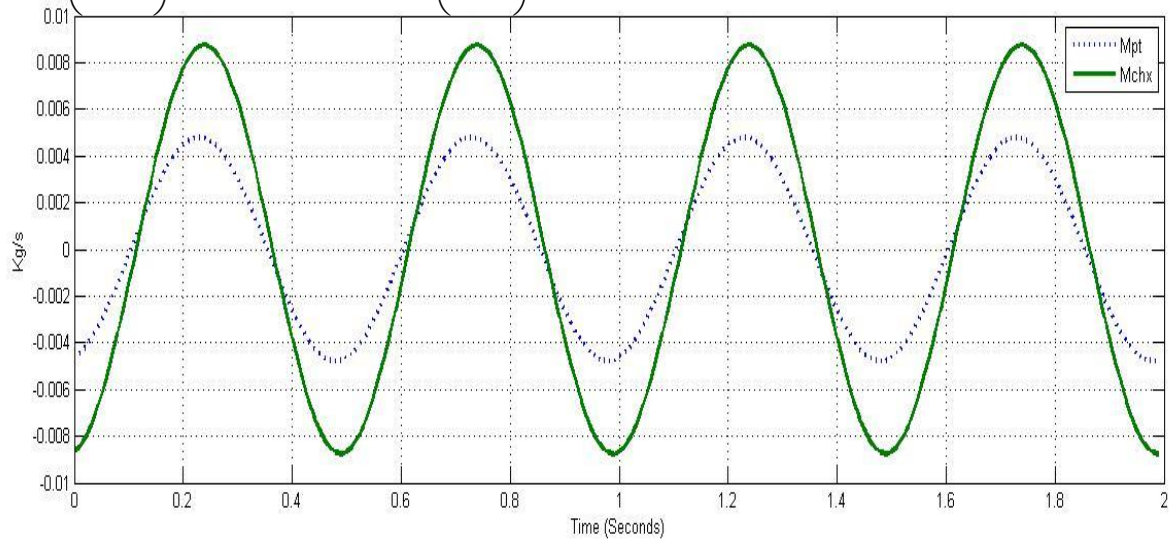


Fig 7.5 Graph representing the mass flow rate through the cold-end heat exchanger $\left(\dot{M}_{chx}\right)$ and the pulse tube $\left(\dot{M}_{pt}\right)$

7.2.2. Data

Table 7.1 Output for the designed consideration

<u>Parameter</u>	<u>Amplitude</u>	<u>Mean Line</u>
Mass flow through orifice	4.1833e-004 kg/s	0 kg/s
Mass flow through hot-end heat exchanger	0.0010 kg/s	0 kg/s
Mass flow through cold-end heat exchanger	0.0088 kg/s	0 kg/s
Mass flow through the pulse tube	0.0048 kg/s	0 kg/s

Mass flow through the regenerator	0.0133 kg/s	0 kg/s
Mass flow through the double inlet valve	0.0018 kg/s	0 kg/s
Mass flow through the compressor	0.0151 kg/s	0 kg/s
Mass flow through the phase	0.0018 kg/s	0 kg/s
Pressure amplitude in the pulse tube	2 MPa	20 MPa
Pressure amplitude in the compressor	2.1760 MPa	20 MPa

Conclusion

A thorough study of the mass flow rate through the various parts of a GM type Double Inlet Pulse Tube Refrigerator (DIPTR) was made. This study helped in developing a MATLAB code which produces the time variation graph for the mass flow rate and pressure oscillation at different parts of the GM type Double Inlet Pulse Tube Refrigerator (DIPTR) when the initial working condition and the dimension for the various component of the DIPTR is provided.

The output of the MATLAB program was further utilized to construct a phasor diagram for the mass flow rates at various sections of the GM type DIPTR, which is a convenient way to observe the phase relationship and hence make necessary adjustment to optimize the output.

At the end of the present work a simplified (approximate) methodology has been put forward for finding out the dimension of the pulse tube of a GM type DIPTR using the phasor diagram whose results were then verified with the outcomes of the MATLAB program.

The output shown by the MATLAB code were within 10% - 15% range of the predicted results which shows that the methodology adopted for designing the pulse tube is considerable to a great extent.

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